Alphametic Puzzles

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April 10, 2006

Alphametic Puzzles

- How can the pattern

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    SEND
+   MORE
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   MONEY
```

- represent a correct sum, if every letter stands for a different decimal digit?

Alphametics

- Term coined by J.A.H. Hunter in 1955
- Also called cryptarithms by S. Vatriquant
- Many alphametic puzzles may be solved by hand – but what if we want to automate the process?

Approach to a solution...

- Step 1: Collect all terms on left-hand-side of the equation
- Example: FL + CA = FUN
- Rewrite as: FL + CA – FUN = 0

Approach to a solution...

- Step 2: treat each letter as an algebraic variable with a value in [0,9] that contributes to the sum or difference by its value and position

- FL = 10*F + 1*L
- CA = 10*C + 1*A
- FUN = 100*F + 10*U + 1*N

Approach to a solution...

- Step 3: solve the simultaneous equations
- Yikes!
- Is there an easier way…?

Key Concept – A Letter's Signature

- Each letter in an alphametic puzzle has a signature that is obtained by substituting 1 for that letter and zero for all the others in the formula
- Represents the contribution this letter makes to the overall answer
- If we multiply all letters' signatures by their values, a correct assignment of values will produce zero

Example Signatures

- F = 10 + 00 –100 = -90
- L = 01 + 00 – 000 = 1
- C = 00 + 10 – 000 = 10
- A = 00 + 01 – 000 = 1
- U = 00 + 00 – 010 = -10
- N = 00 + 00 – 001 = -1

So….?

- The problem now is to find all permutations $a_1 \ldots a_{10}$ of {0, 1, 2, …, 9} such that $a_1 s_1 + a_2 s_2 + \ldots + a_{10} s_{10} = 0$
- So let's generate all 10! Permutations and try each one to see if we have a solution!

Java to the Rescue!

- 10! would be a large number of possibilities to try by hand!
- Let's see a computer implementation of this idea:

- http://www.mountunion.edu/~cindricbb/sp06/alphametics/TheApplet.html

Reference

- Knuth, Donald. The Art of Computer Programming, Vol. 4, Fascicle 2, Addison-Wesley, 2005, pp. 44-45.